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SHIP CONTROL AND MATHEMATICAL MODELING OF ITS MOVEMENT

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Abstract. *The most pressing challenges of the modern world are environmental issues, rising energy and food prices. In these conditions, research is becoming increasingly important to minimise shipping costs, optimise routes, and explore issues of improving shipping safety. Equally important are the issues of organising search and rescue operations and maritime military operations. Mathematical modelling is a powerful tool for solving these and many related problems. Methods of mathematical analysis, optimal control theory and differential equations are widely used in the construction and research of mathematical models. The main tasks of this work are to study the issues of route optimisation, ship speed control and selection of the most economical power plant operation mode. Various aspects of mathematical modelling of ship motion are considered. The issue of ship speed control in variable navigation conditions is studied using optimal control methods. The task is to find the law of change in the ship's speed, which ensures minimum fuel consumption during a given length of the race. The paper also considers the problem of choosing the trajectory and mode of operation of the power plant, which together ensure the arrival of the vessel at the destination at a given time with minimal fuel consumption. Differential models of ship motion are developed and studied, in particular, the issue of choosing the optimal trajectory of a marine drone for the fastest possible target destruction is considered. The results obtained can be used to develop a method of ship control, in which the task will be completed on time and at minimal cost. The study of the problems considered in this article can and should be an integral part of the mathematical training of specialists in the maritime industry.*

Key words: *optimal trajectory, total cost, conditional extremum, optimal control, differential equations.*

Сиваш С. Б., Соколовська Г. В. КЕРУВАННЯ СУДНОМ ТА МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ЙОГО РУХУ

Анотація. *Найактуальнішими викликами сучасного світу є проблеми екології, удорожчання енергоресурсів та продовольства. У цих умовах зростає актуальність наукових досліджень, що мають на меті мінімізувати витрати на морські перевезення, оптимізувати маршрути, дослідити питання підвищення безпеки судноплавства. Не менш важливими є питання організації пошуково-рятувальних операцій та морських військових операцій. Математичне моделювання є потужним інструментом у розв'язуванні цих та багатьох суміжних задач. При побудові та дослідженні математичних моделей широко використовуються методи математичного аналізу, теорія оптимального керування та диференціальних рівнянь. Головними задачами даної роботи є дослідження питань оптимізації маршрутів, керування швидкістю судна та вибором найбільш економічного режиму роботи силової установки. Розглянуто різноманітні аспекти математичного моделювання руху судна. Методами оптимального керування досліджується питання контролю швидкості судна у змінних умовах плавання. Розглядається задача: знайти закон зміни швидкості руху судна, який забезпечує мінімум витрат палива при проходженні перегону заданої довжини. Розглянуто також задачу вибору траєкторії та режиму роботи силової установки, які в сукупності забезпечують прибуття судна до пункту призначення у заданий момент часу при мінімальних витратах пального. Складено та досліджено диференціальні моделі руху судна, зокрема розглянуто питання вибору оптимальної траєкторії руху морського дрона для якнайшвидшого ураження цілі. Отримані результати можуть бути використані при розробці методу керування судном, при якому* *поставлене завдання буде виконано вчасно і за мінімальних витрат. Вивчення задач, що розглядаються у статті, може і повинно бути складовою частиною математичної підготовки фахівців у морській галузі.*

Ключові слова: *оптимальна траєкторія, загальні витрати, умовний екстремум, оптимальне керування, диференціальні рівняння.*

Problem statement. With the cost of fuels and lubricants constantly rising, ship traffic management methods need to be improved. The main tasks of this process are to select a safe, time-optimised route and the appropriate mode of propulsion. These issues are of particular importance during search and rescue operations and maritime military operations.

Background and Literature Review. Optimal control theory [1] and mathematical modelling [2] have been the subject of research by numerous scientists. Paper [3] discusses mathematical models of some water engineering problems. Many scientists have studied the issues of improving methods of ship traffic control. Paper [4] addresses the issue of developing routes for vessels with due regard for environmental safety. Paper [5] considers methods of monitoring and controlling the movement of ships during manoeuvring to ensure their safe separation under certain conditions. The issues of improving automatic ship traffic control systems are addressed in [6]. It considers the method of forming a safe, time-optimal ship route taking into account the forecast of weather conditions. Numerous scientific studies are devoted to the improvement of automated ship control systems through the use of artificial intelligence. Paper [7] investigates data recognition methods in shipping to build a trajectory for an unmanned ship with artificial intelligence. Remote vessel control technologies and theoretical aspects of the introduction of autonomous navigation are studied in [8]. The authors of [9] emphasise the prospects of using unmanned aerial vehicles in modern shipping. The paper explores various aspects of drone integration into maritime operations to improve communication, data collection, etc. The use of drones for ship maintenance and inspections can save time and money. The issue of time is particularly acute in maritime rescue operations [10]. In such situations, it is extremely important to choose the optimal and safe route for the vessel.

Summary of the main material. Problem 1 (controlling the vessel's speed in variable navigation conditions). We consider the water resistance to the ship's motion F proportional (in relative units) to the square of the speed, and the power of the propulsion system N – to the cube of the speed. Let S be the distance and v the speed. Then

$$F=\gamma_1 v^2=\gamma_1\left(\frac{dS}{dt}\right)^2;\ N=\gamma_2 v^3=\gamma_2\left(\frac{dS}{dt}\right)^3.$$

We also assume that the intensity q of fuel consumption by the power plant (measured in kg/h) is proportional to the cube of the speed: $q=\gamma v^3=\gamma\left(\frac{dS}{dt}\right)^3$. In general, the values γ, γ_1, γ_2 are functions and depend on the current coordinates, vessel load, etc. Let us assume that they are functions of S:

$$q=\gamma v^3=\gamma(S)\left(\frac{dS}{dt}\right)^3 \qquad (1)$$

Let's express the total fuel consumption G for a race of length S_0 in time T:

$$G=\int_0^T q\,dt=\int_0^T \gamma(S)\left(\frac{dS}{dt}\right)^3 dt. \qquad (2)$$

Let's formulate the control problem. Find the law of change of the vessel's speed v, which ensures the minimum fuel consumption during the passage of a race of length S_0 in time T in comparison with all other modes of movement along the same trajectory. In other words, it is necessary to find $S(t)$ or $v(t)$ at $S(0)=0$; $S(T)=S_0$, which ensures the minimum of the integral (2). According to [2], the function $q(x)$ must satisfy the Euler's equation. In this case, the Euler's equation has the form:

$$F-y'F'_{y'}=c \text{ or } y'F'_{y'}-F=c, \qquad (3)$$

where $F=q=\gamma(S)\left(\frac{dS}{dt}\right)^3$, $F'_{y'}=3\gamma(S)\left(\frac{dS}{dt}\right)^2$. (4)

Then $q-3q=c;\ q=c_1=const$. It follows that the fuel consumption should be constant. From equations (3), (4), we express the speed:

$$2\gamma v^3=c;\ v=\sqrt[3]{\frac{c}{2\gamma(S)}}.$$

Thus, the speed of the vessel is determined by the law of distribution of the coefficient γ(*S*) along the route. The function γ(*S*) depends mainly on the draft, i.e., the load. To determine the correct sailing speed, you need to know the predicted load ahead on the route. The function γ(*S*) and the corresponding optimal control can then be developed.

The law of fuel consumption can be determined by the variational method in the case of generalisation of condition (1): $q=q\left(S,\frac{dS}{dt}\right)=q(S,v)$. Then the integral (2) takes the form $G=\int_0^T q\left(S,\frac{dS}{dt}\right)dt=\min$. The Euler's equation $\frac{\partial q}{\partial S}-\frac{d}{dt}\cdot\frac{\partial q}{\partial v}=0$ becomes the equation $v\cdot\frac{\partial q}{\partial v}-q=c$, where *c* is determined from the condition $S(T)=S_0$.

Importantly, these results can be applied to any vehicle.

Problem 2. Consider the problem of choosing the trajectory and the mode of operation of the power plant, which together ensure the arrival of the vessel at the destination at a given time with minimal fuel consumption.

The motion takes place in the *xy*-plane. Let the point of departure coincide with the origin and the point of arrival be at $B(x_0, y_0)$. The velocity of the flow $\overline{v_T}(x,y)$ at each point of the trajectory and its projections on the coordinate axes $v_{Tx}(x, y)$; $v_{Ty}(x, y)$ are known. The speed $\bar{v}$ of the vessel is the geometric sum of the flow speed $\bar{v}_T$ and the vessel's speed $\overline{v_0}$ relative to the water: $\overline{v}=\overline{v_0}+\overline{v_T}$;

$$v_x=v_{0x}+v_{Tx}. \tag{5}$$

Let the velocity of the ship relative to the water form an angle α with the axis *Ox*. Then $v_{0x}=v_0\cos\alpha$; $v_{0y}=v_0\sin\alpha$.

Consider that $v'_x=\frac{dx}{dt}$; $v'_y=\frac{dy}{dt}$. Then from equation (5) we obtain:

$$\frac{dx}{dt}=v_0\cos\alpha+v_{xT};\quad \frac{dy}{dt}=v_0\sin\alpha+v_{yT};$$

$$\frac{dx}{v_0\cos\alpha+v_{xT}}=dt;\quad \frac{dy}{v_0\sin\alpha+v_{yT}}=dt.$$

Vessel's time of movement to point *B*

$$T=\int_0^{x_0}\frac{dx}{v_0\cos\alpha+v_{xT}}. \tag{6}$$

The condition for his arrival at this point is: $y=y_0$. We assume that

$$\frac{dy}{dx}=\frac{v_0\sin\alpha+v_{yT}}{v_0\cos\alpha+v_{xT}};\quad dy=\frac{v_0\sin\alpha+v_{yT}}{v_0\cos\alpha+v_{xT}}dx.$$

Then we get:

$$y=\int_0^{x_0}\frac{v_0\sin\alpha+v_{yT}}{v_0\cos\alpha+v_{xT}}dx. \tag{7}$$

Let's set the problem of determining the optimal trajectory α(*x*) that ensures the minimum of the integral (6) under the condition (7). This is a conditional extremum problem. The Lagrange functions

$$F=\frac{1}{v_0\cos\alpha+v_{xT}}+\lambda_0\frac{v_0\sin\alpha+v_{yT}}{v_0\cos\alpha+v_{xT}}.$$

The Euler-Lagrange equation

$$\frac{\partial F}{\partial\alpha}=\frac{v_0\sin\alpha}{\left(v_0\cos\alpha+v_{xT}\right)^2}+$$

$$+\lambda_0\frac{v_0\cos\alpha\left(v_0\cos\alpha+v_{xT}\right)+v_0\sin\alpha\left(v_0\sin\alpha+v_{yT}\right)}{\left(v_0\cos\alpha+v_{xT}\right)^2};$$

$$\frac{\partial F}{\partial\alpha}=\frac{v_0\sin\alpha+\lambda_0 v_0{}^2+v_0\left(v_{xT}\cos\alpha+v_{yT}\sin\alpha\right)\lambda_0}{\left(v_0\cos\alpha+v_{xT}\right)^2};$$

$$\frac{\partial F}{\partial\alpha'}=0.$$

This equation is algebraic:

$$\frac{v_0}{\sin\alpha}+v_{xT}\cdot ctg\alpha+v_{yT}=-\frac{1}{\lambda_0}. \tag{8}$$

From this equation, we should find α(*x*) provided that $0\le\alpha(x)\le\frac{\pi}{2}$.

If $v_{xT}=v_{yT}=0$, then from equation (8) we obtain: $\frac{1}{\sin\alpha}=-\frac{1}{\lambda_0 v_0}-\frac{v_{yT}}{v_0}$.

From this equation, it follows that the straight-line path will no longer be optimal. The value λ_0 is determined from condition (7).

Now let's set the task of minimising fuel consumption, given the time of movement *T* to a given point $B(x_0, y_0)$. To calculate the fuel consumption, we use formula (1). Let $\gamma=const=1$.

Then $q = v_0^3$. Total expenses

$$G = \int_0^T q dt = \int_0^T v_0^3 dt\,. \tag{9}$$

Thus, the problem is to find the functions α(*x*), $v_0(x)$ so that the integral (9) becomes the smallest value under the conditions (6), (7). This is a conditional extremum problem with integral relations (6), (7). The integral (9) can be written in the form

$$G = \int_0^T v_0^3 dt = \int_0^{x_0} \frac{v_0^3}{v_0 \cos\alpha + v_{xT}} dx. \tag{10}$$

Let's construct the Lagrange function:

$$F = \frac{v_0^3}{v_0 \cos\alpha + v_{xT}} + \lambda_1 \frac{1}{v_0 \cos\alpha + v_{xT}} + \lambda_2 \frac{v_0 \sin\alpha + v_{yT}}{v_0 \cos\alpha + v_{xT}}. \tag{11}$$

The Euler-Lagrange equations for functions α(*x*) and $v_0(x)$ will also be algebraic:

$$\frac{\partial \alpha(x)}{\partial t} = 3v_0^3 ctg\alpha + 3v_0^2 v_{xT} \frac{1}{\sin\alpha} + \frac{v_0^3}{\sin\alpha} -$$

$$-\lambda_1 ctg\alpha + \lambda_2 v_{xT} - \lambda_2 v_{yT} ctg\alpha = 0; \tag{12}$$

$$\frac{\partial v_0(x)}{\partial t} = v_0^3 + \lambda_1 + \lambda_2 v_0 \frac{1}{\sin\alpha} + \lambda_2 v_{xT} ctg\alpha + v_{yT} = 0. \tag{13}$$

From the resulting system of equations, we can find α(*x*) and $v_0(x)$. To equations (12), (13), we should add conditions (6), (7), from which we can find λ_1, λ_2. Note that this system can be solved only by numerical methods.

Problem 3. The vessel leaves point *O* at a constant speed and sails in the direction of the *Oy*-axis (Fig. 1). At the same time, a marine drone starts from point *A*, located at a distance *a* from the vessel, to catch up with it. The speed of the drone is twice the speed of the vessel. Find the equation of the drone's trajectory (chase curve) and the minimum time it takes to hit the target.

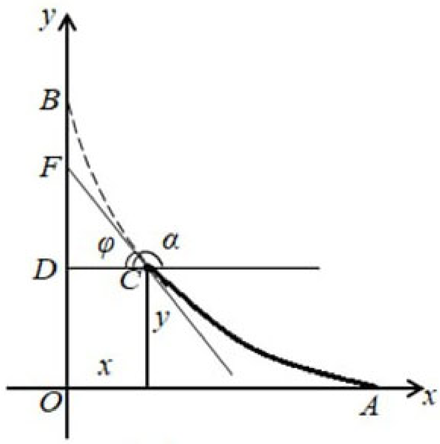

Fig. 1. The trajectory of a marine drone

Let the drone be at point *C*(*x; y*) and the ship at point *F* at time *t*. The problem is to find the equation of the trajectory, which the drone should follow so that the tangent line crosses the *Oy*-axis at *F*.

By this time, the vessel has already travelled the path *OF* = *vt*, *l* is the length of the arc *AC* of the drone's trajectory. Let us find the angle of inclination of its tangent *CF* to the positive direction of the axis *Ox*:

$$\frac{dy}{dx} = tg\alpha = -tg\varphi = -\frac{FD}{CD} = -\frac{vt - y}{x} = \frac{y - vt}{x}.$$

On the other hand, $l = 2vt$. So $\frac{dy}{dx} = \frac{y - v\frac{l}{2v}}{x}$ or $x\frac{dy}{dx} - y = -\frac{l}{2}$. Let's differentiate both parts of the equation $\frac{dy}{dx} + \tilde{o}\frac{d^2y}{dx^2} - \frac{dy}{dx} = -\frac{1}{2}\frac{dl}{dx}$.

Let's use the formula to calculate the arc length differential:

$$dl = -\sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx\,.$$

The minus in this formula is due to the fact that the movement along the curve is in the direction of decreasing *x*. We get a second-order differential equation: $x\frac{d^2y}{dx^2} = \frac{1}{2}\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$.

Let's reduce its order by replacing the unknown function: $\frac{dy}{dx} = p(x)$, $\frac{d^2y}{dx^2} = \frac{dp}{dx}$.

The equation takes on the form: $x\frac{dp}{dx} = \frac{1}{2}\sqrt{1 + p^2}$.

Separate the variables and integrate. We have: $\ln\left|p + \sqrt{1 + p^2}\right| = \frac{1}{2}\ln|x| + C$ or $p + \sqrt{1 + p^2} = C\sqrt{x}$ (*C* – arbitrary constant).

Note that at the initial time (*t* = 0): $x = a,\ y(a) = 0,\ p(a) = \left.\frac{dy}{dx}\right|_{x=a} = 0.$ Let's substitute the initial conditions and get it:

$$0 + \sqrt{1 + 0^2} = C\sqrt{a} \Rightarrow C = \frac{1}{\sqrt{a}},$$

$$p+\sqrt{1+p^2}=\sqrt{\frac{x}{a}}. \quad (14)$$

Let's transform the equation (14).

$$\frac{\left(p+\sqrt{1+p^2}\right)\left(p-\sqrt{1+p^2}\right)}{p-\sqrt{1+p^2}}=\sqrt{\frac{x}{a}} \Leftrightarrow \frac{-1}{p-\sqrt{1+p^2}}=\sqrt{\frac{x}{a}}.$$

Therefore:

$$p-\sqrt{1+p^2}=-\sqrt{\frac{a}{x}}. \quad (15)$$

Adding equations (14) and (15), we obtain:

$$2p=\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}} \text{ or } \frac{dy}{dx}=\frac{1}{2}\left(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}}\right).$$

Let's solve this equation:

$$y=\frac{1}{2}\int\left(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}}\right)dx \Leftrightarrow y=\frac{x\sqrt{x}}{3\sqrt{a}}-\sqrt{ax}+C,$$

C – arbitrary constant. From the initial condition $y(a)=0$, we obtain: $C=\frac{2}{3}a$. Thus, the equation of the drone's trajectory is as follows:

$$y=\frac{x\sqrt{x}-3a\sqrt{x}+2a\sqrt{a}}{3\sqrt{a}}.$$

The drone will catch up with the vessel at the moment when $x = 0$. By substituting zero for x, we obtain $y=\frac{2}{3}a$.. This is the distance the ship will travel before meeting the drone. So, the time you are looking for is $t=\frac{y}{v}=\frac{2a}{3v}$.

Problem 4. A vessel with displacement $P = 1500t$, overcomes the water resistance force $R = \alpha v^2$, $\alpha = 0{,}12$, m – mass of the vessel, v – its speed. Screw traction force is equal to $T=T_0\left(1-\frac{v}{v_s}\right)$ (T_0 = 120 т), $v_s = const$ = 33 m/s. Find the dependences of:

1) vessel's speed on time, if the initial speed is equal to v_0;

2) the distance travelled on the speed;

3) distance on time at the initial speed v_0= 10 m/s.

Let x be the distance travelled, then $v=\frac{dx}{dt}$ is the speed and $a=\frac{d^2x}{dt^2}$ is the acceleration. By projecting all the acting forces horizontally, taking into account their directions, we obtain a second-order differential equation:

$$m\frac{d^2x}{dt^2}+\alpha\left(\frac{dx}{dt}\right)^2-T_0\left(1-\frac{1}{v_s}\frac{dx}{dt}\right)=0$$

or first-order equation:

$$m\frac{dv}{dt}+\alpha v^2-T_0\left(1-\frac{v}{v_s}\right)=0. \quad (16)$$

Separate the variables and integrate both parts of the equation

$$-\frac{m}{\alpha}\int\frac{dv}{v^2+\frac{vT_0}{\alpha v_s}-\frac{T_0}{\alpha}}=\int dt,$$

$$-\frac{m}{2\sqrt{T_0\left(\alpha+\frac{T_0}{4v_s^2}\right)}}\ln\left|\frac{\alpha v+\frac{T_0}{2v_s}-\sqrt{T_0\left(\alpha+\frac{T_0}{4v_s^2}\right)}}{\alpha v+\frac{T_0}{2v_s}+\sqrt{T_0\left(\alpha+\frac{T_0}{4v_s^2}\right)}}\right|=t+C,$$

C is arbitrary constant. Enter the designation: $\sqrt{T_0\left(\alpha+\frac{T_0}{4v_s^2}\right)}=\beta$. Let's use the initial conditions $v = v_0$ if t = 0. We obtain:

$$C=-\frac{m}{2\beta}\ln\left|\frac{\alpha v_0+\frac{T_0}{2v_s}-\beta}{\alpha v_0+\frac{T_0}{2v_s}+\beta}\right|$$

The solution of the Cauchy problem is

$$t=\frac{m}{2\beta}\ln\left|\frac{\left(\alpha v+\frac{T_0}{2v_s}+\beta\right)\left(\alpha v_0+\frac{T_0}{2v_s}-\beta\right)}{\left(\alpha v+\frac{T_0}{2v_s}-\beta\right)\left(\alpha v_0+\frac{T_0}{2v_s}+\beta\right)}\right|.$$

According to the task conditions $\beta=4{,}2,\ \frac{T_0}{2v_s}=1{,}8,\ \frac{2\beta}{m}=0{,}055.$

Then the required solution can be written in the form: $\frac{(v+50)(v_0-20)}{(v-20)(v_0+50)}=e^{0{,}055t}$. Let us express v from this equation

$$v=\frac{70v_0+20(v_0+50)\left(e^{0{,}055t}-1\right)}{70+(v_0+50)\left(e^{0{,}055t}-1\right)}. \quad (17)$$

To find the dependence of the distance travelled on the speed, we need to write equation (16) in the form

$$\frac{mdv}{\alpha v^2+\frac{T_0 v}{v_s}-T_0}=-dt.$$

Then we should multiply both parts of the equation by v. The equation takes the form

$$\frac{mvdv}{\alpha v^2+\frac{T_0 v}{v_s}-T_0}=-dx$$

(because $vdt=\frac{dx}{dt}dt=dx$). The general solution of the equation is

$$-x+C=\frac{m}{2\alpha}\ln\left|\left(\frac{v+\gamma}{\sqrt{\delta}}\right)^2-1\right|+\frac{m\gamma}{2\alpha\sqrt{\delta}}\ln\left|\frac{v+\gamma+\sqrt{\delta}}{v+\gamma-\sqrt{\delta}}\right|,$$

where $\gamma=\frac{T_0}{2\alpha v_s}=15$ m/s, $\delta=\frac{T_0}{\alpha}\left(1+\frac{T_0}{4\alpha v_s^2}\right)=1225$ m/s.

Let's set the initial conditions $x|_{t=0}=0$, $v|_{t=0}=v_0$. We obtain:

$$C=\frac{m}{2\alpha}\ln\left|\left(\frac{v_0+\gamma}{\sqrt{\delta}}\right)^2-1\right|+\frac{m\gamma}{2\alpha\sqrt{\delta}}\ln\left|\frac{v_0+\gamma+\sqrt{\delta}}{v_0+\gamma-\sqrt{\delta}}\right|$$

or

$$C=637{,}5\ln\left|\left(\frac{v_0+15}{35}\right)^2-1\right|+273{,}9\ln\left|\frac{v_0+50}{v_0-20}\right|.$$

Thus, the dependence of the distance travelled on the speed is defined by the equality

$$x=637{,}5\ln\left|\frac{(v_0+15)^2-1225}{(v+15)^2-1225}\right|+273{,}9\ln\left|\frac{(v_0+50)(v-20)}{(v_0-20)(v+50)}\right|.$$

Now let's find the dependence of the distance travelled on time at the initial speed v_0 = 10 m/s. Equation (17) can be written in the form

$$dx=\frac{70v_0+20(v_0+50)\left(e^{0,055t}-1\right)}{70+(v_0+50)\left(e^{0,055t}-1\right)}dt=\frac{1200e^{0,055t}-50}{60e^{0,055t}+10}dt.$$

Let's integrate it

$$x+C=\int\left(20-70\cdot\frac{1}{6e^{0,055t}+1}\right)dt;$$

$$x+C=20t-70\int\frac{dt}{6e^{0,055t}+1}.$$

In this case, we need to replace the variable using the formula $u=e^{0,055t}$. So, we have:

$$x+C=20t-\frac{70}{0,055}\ln\frac{e^{0,055t}}{e^{0,055t}+\frac{1}{6}}.$$

From the initial condition $x|_{t=0}=0$, we find $C=-\frac{70}{0,055}\ln\frac{6}{7}$. Thus, the path dependence on time is as follows

$$x=20t+\frac{70}{0,055}\ln\frac{6e^{0,055t}+1}{7e^{0,055t}}.$$

Conclusions. The results obtained can be used to select the most efficient method of ship control. The development of ship navigation technologies, in particular remote-control systems, requires high-quality mathematical education from specialists. The problems discussed in this article can be included as applied problems in the course of higher mathematics at a technical university.

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